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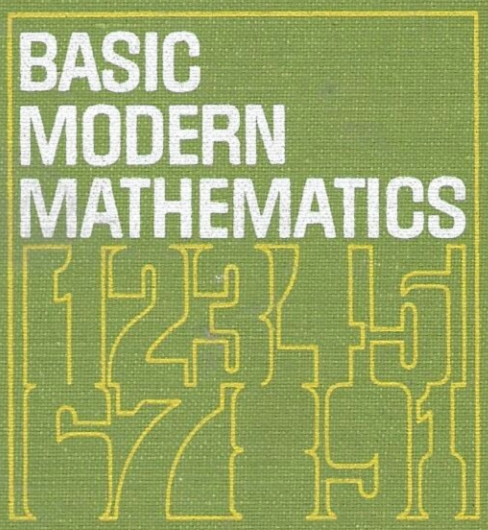

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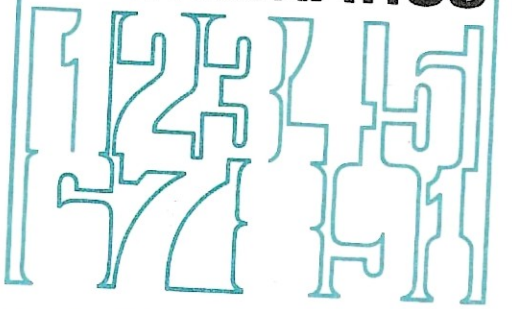

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Chapter 1 PLACE VALUE

Grouping by tens

Using place value, we can write the numeral for any whole number with only the digits

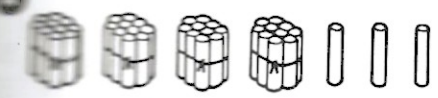
0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

A digit may stand for different numbers. What the digit stands for depends on its place or position in the numeral. Here is an example.

2 3 4 5 ⟶ The 5 stands for 5 ones (5).
4 2 5 3 ⟶ The 5 stands for 5 tens (50).
3 5 4 2 ⟶ The 5 stands for 5 hundreds (500).
5 3 2 4 ⟶ The 5 stands for 5 thousands (5000).

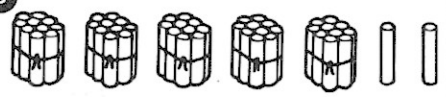
Here are some examples to help you think about the grouping by tens in the place-value system.

A



4 tens and 3
We write 43.

B



5 tens and 2
We write 52.

EXERCISES

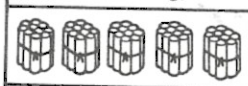
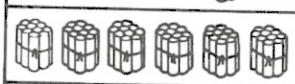
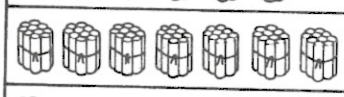
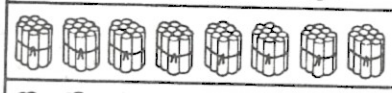
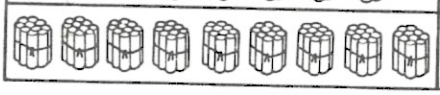
1. Write the 2-digit numeral for each of these.

[A] 2 tens and 3
[B] 4 tens and 2
[C] 6 tens and 7
[D] 9 tens and 3
[E] 6 tens and 0
[F] 1 ten and 1

2. Give the correct digit for each ▒.

[A] 37 means 3 tens and ▒.
[B] 48 means ▒ tens and 8.
[C] 39 means 3 tens and ▒.
[D] 15 means ▒ tens and 5.

2-digit numerals

We see	We think	We write	We say
	2 tens	20	twenty
	3 tens	30	thirty
	4 tens	40	forty
	5 tens	50	fifty
	6 tens	60	sixty
	7 tens	70	seventy
	8 tens	80	eighty
	9 tens	90	ninety

ORAL EXERCISES

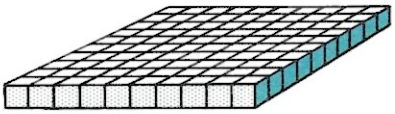
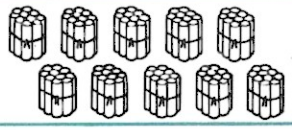
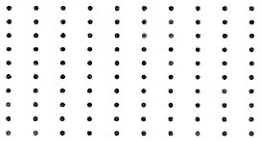
1. Read the number. Then tell how many tens and how many ones.

[A] 63 [B] 47 [C] 39 [D] 57 [E] 21 [F] 16 [G] 96 [H] 85 [I] 43 [J] 60 [K] 51 [L] 30 [M] 17 [N] 21 [O] 78 [P] 11 [Q] 10 [R] 99 [S] 48 [T] 80 [U] 27 [V] 50 [W] 35 [X] 70

2. Give the correct number for each of these.

[A] 6 tens and 5
[B] 5 tens and 9
[C] 4 tens and 2
[D] 5 tens and 4
[E] 6 tens and 2
[F] 3 tens and 0
[G] 1 ten and 3
[H] 4 tens and 6
[I] 8 tens and 3
[J] 3 tens and 6
[K] 4 tens and 3
[L] 1 ten and 0
[M] 2 tens and 9
[N] 7 tens and 1
[O] 7 tens and 7
[P] 6 tens and 0
[Q] 7 tens and 0
[R] 4 tens and 9
[S] 8 tens and 5
[T] 7 tens and 3
[U] 9 tens and 2
[V] 1 ten and 5
[W] 8 tens and 0
[X] 6 tens and 8

Hundreds

Ten	10 tens
	one hundred 
	one hundred 
	one hundred 

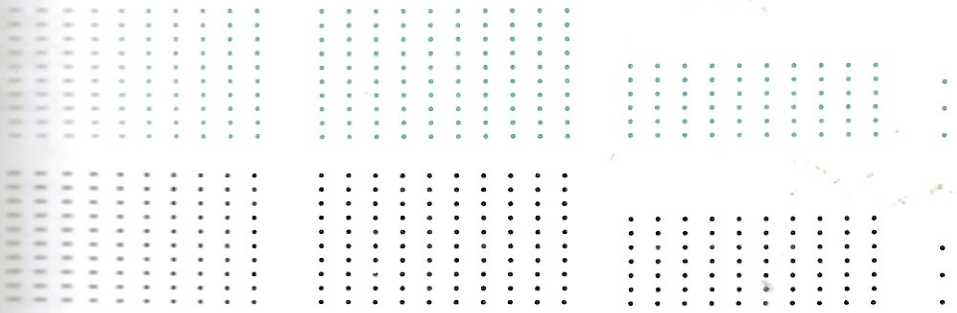
EXERCISES

1. Draw a set of one hundred dots (10 rows of ten).
   Now draw a set of thirty dots (3 rows of ten).
   Now draw a set of seven dots.
   You have drawn one hundred thirty-seven dots.

2. Did you need to count higher than 10 to draw the dots in exercise 1?

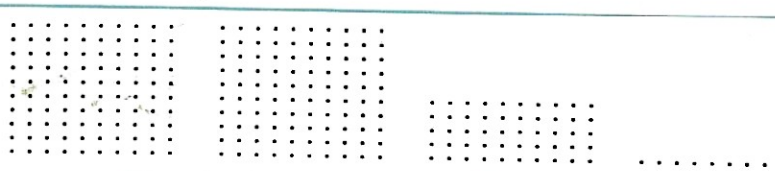
3. Draw a set of one hundred twenty-three dots.
   Do not count higher than 10.

4. Which set of dots has more, the blue or the black?



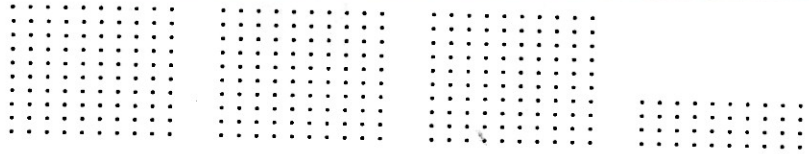
5. How many more does one set have than the other?

3-digit numerals



2 hundreds 5 tens 8 ones

We write 258.



3 hundreds 4 tens 5 ones

We write 345.

ORAL EXERCISES

Read the number. Then tell how many hundreds, tens, and ones.

1. 278	**4.** 923	**7.** 318	**10.** 704	**13.** 707	**16.** 864
2. 346	**5.** 765	**8.** 640	**11.** 900	**14.** 770	**17.** 297
3. 512	**6.** 492	**9.** 380	**12.** 506	**15.** 437	**18.** 485

EXERCISES

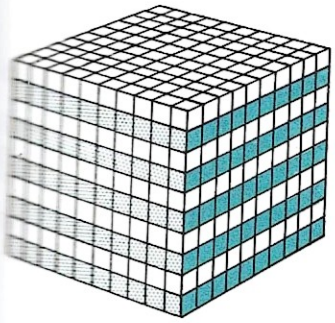
1. Write the numeral. (*h* stands for hundreds and *t* for tens.)

[A] 3 *h*, 2 *t*, and 6
[B] 4 *h*, 7 *t*, and 2
[C] 6 *h*, 5 *t*, and 0
[D] 6 *h*, 0 *t*, and 1
[E] 3 *h*, 2 *t*, and 0
[F] 3 *h*, 0 *t*, and 0
[G] 6 *h*, 2 *t*, and 7
[H] 1 *h*, 0 *t*, and 0
[I] 7 *h*, 6 *t*, and 5

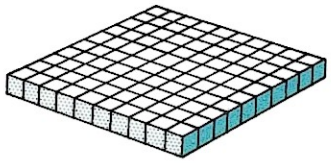
2. Give the missing digit.

[A] 384 means 3 hundreds, ▮ tens, and 4 ones.
[B] 659 means ▮ hundreds, 5 tens, and 9 ones.
[C] 518 means 5 hundreds, 1 ten, and ▮ ones.
[D] 304 means ▮ hundreds, 0 tens, and 4 ones.
[E] 760 means 7 hundreds, 6 tens, and ▮ ones.
[F] 927 means 9 hundreds, ▮ tens, and 7 ones.

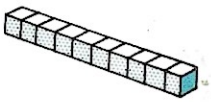
Thousands



10 hundreds
one thousand



10 tens
one hundred



ten

DISCUSSION EXERCISES

1. Can you draw a thousand dots without counting higher than 10? Explain how to do this.
2. How many tens are in one thousand? Explain how you found out.

For 1 thousand, 3 hundreds, 2 tens, and 6, we write 1326.

For 6 thousands, 4 hundreds, 7 tens, and 5, we write 6475.

For 8 thousands, 2 hundreds, 0 tens, and 7, we write 8207.

ORAL EXERCISES

Read the number. Then tell how many thousands, hundreds, tens, and ones.

1. 7264
2. 8315
3. 9126
4. 8427
5. 1635
6. 7986
7. 8204
8. 3716
9. 9025
10. 8840
11. 7602
12. 9100
13. 8340
14. 2600
15. 5000
16. 8083
17. 9216
18. 7007
19. 6000
20. 5080
21. 3210
22. 1000
23. 9020
24. 8003

Place-value exercises

1. Write the 4-digit numeral for each of these. (*th* stands for thousands, *h* stands for hundreds, and *t* stands for tens.)

[A] 6 *th*, 5 *h*, 2 *t*, and 4
[B] 1 *th*, 2 *h*, 3 *t*, and 9
[C] 9 *th*, 4 *h*, 2 *t*, and 1
[D] 9 *th*, 4 *h*, 2 *t*, and 0
[E] 9 *th*, 4 *h*, 0 *t*, and 0
[F] 9 *th*, 0 *h*, 0 *t*, and 0
[G] 1 *th*, 1 *h*, 1 *t*, and 1
[H] 1 *th*, 0 *h*, 0 *t*, and 0
[I] 6 *th*, 0 *h*, 8 *t*, and 0
[J] 4 *th*, 5 *h*, 0 *t*, and 2
[K] 7 *th*, 3 *h*, 5 *t*, and 6
[L] 3 *th*, 2 *h*, 0 *t*, and 0

2. The numerals 6325 and 6425 differ only in the hundreds place. Which number, 6325 or 6425, has more hundreds? Which of the two numbers is larger? How much larger is one than the other?

3. Give the number that is 10 more than

[A] 25
[B] 68
[C] 468
[D] 725
[E] 841
[F] 1425
[G] 3168
[H] 9256
[I] 595
[J] 307

4. Give the numbers that are 10 less than those in exercise 3.

5. Give the number that is 100 more than

[A] 256
[B] 682
[C] 4682
[D] 7256
[E] 8417
[F] 1425
[G] 3168
[H] 3426
[I] 2927
[J] 4036

6. Give the numbers that are 100 less than those in exercise 5.

7. What number is 10 more than 5? What number is 100 more than 5? What number is 1000 more than 5?

8. From each pair of numbers, choose the larger.

[A] 6254 or 6354
[B] 8321 or 7321
[C] 5280 or 5281
[D] 1728 or 1720
[E] 3256 or 3259
[F] 2999 or 3000
[G] 4321 or 4391
[H] 7192 or 7092
[I] 5300 or 5200
[J] 4200 or 4070
[K] 1283 or 1279
[L] 3260 or 3209

9. Rearrange these numbers from smallest to largest.

[A] 689, 52, 1683, 100
[B] 3429, 516, 8154, 275
[C] 5654, 5454, 5154, 5954
[D] 8276, 8216, 8246, 8236
[E] 7920, 7918, 7921, 7919
[F] 5278, 5280, 5279, 5281
[G] 6498, 6500, 6499, 6501
[H] 3999, 3998, 4000, 4001

10. Give the next 3 numbers for each of these sequences.

[A] 289, 290, 291, . . .
[B] 566, 567, 568, . . .
[C] 1298, 1299, 1300, . . .
[D] 4380, 4390, 4400, . . .
[E] 7256, 7356, 7456, . . .
[F] 3173, 3183, 3193, . . .
[G] 6496, 6497, 6498, . . .
[H] 2995, 2996, 2997, . . .

11. In each numeral below, one of the digits is blue. Give the number for which that digit stands. For example, in exercise A the 7 stands for 700.

[A] 6728
[B] 4325
[C] 4286
[D] 9515
[E] 7106
[F] 8732
[G] 9457
[H] 1260
[I] 4037
[J] 5208

12. Find the missing digits for each of these.

[A] 6721 means ▮ hundreds, 6 thousands, 1 one, ▮ tens.
[B] 4362 means 4 thousands, ▮ tens, ▮ hundreds, 2 ones.
[C] 7820 means 2 tens, ▮ hundreds, 0 ones, ▮ thousands.
[D] 5207 means ▮ hundreds, ▮ tens, 5 thousands, 7 ones.
[E] 6431 means 1 one, 6 thousands, ▮ tens, ▮ hundreds.
[F] 2803 means 2 thousands, ▮ tens, ▮ hundreds, 3 ones.
[G] 4076 means ▮ tens, 0 hundreds, ▮ ones, 4 thousands.
[H] 8030 means 0 ones, ▮ thousands, ▮ hundreds, 3 tens.

★13. Think about the number 8253. Now answer these questions.

[A] What number is 100 and 10 more than 8253?
[B] What number is 1000 and 3 less than 8253?
[C] What number is 253 less than 8253?
[D] What number is 8000 less than 8253?
[E] What number is 8000 and 50 less than 8253?

Reading the odometer

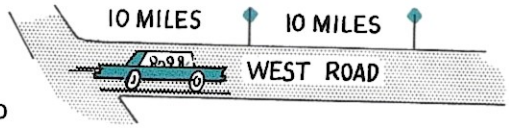
EXERCISES

1. When Jim's family left home, the odometer on their car looked like this. → 9765

Jim saw this sign.

What did the odometer read when they were in Redtown?

2. West Road had markers every 10 miles. When the car turned onto West Road, the odometer looked like this. → 10240



Jim counted 10 markers. He surprised his sister Nancy by telling her what the number on the odometer was without looking. What did the odometer read?

3. At Wood City the odometer looked like this. → 10784

At River City the odometer looked like this. → 10794

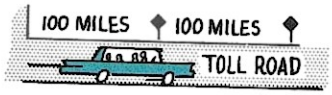
How far is it from Wood City to River City?

4. Logtown is 100 miles farther from Jim's house than Fish Hook is.

The odometer looked like this at Fish Hook. → 11065

What do you think the odometer read at Logtown?

5. The toll road had black markers every 100 miles. When the car turned onto the toll road the odometer looked like this. → 12043

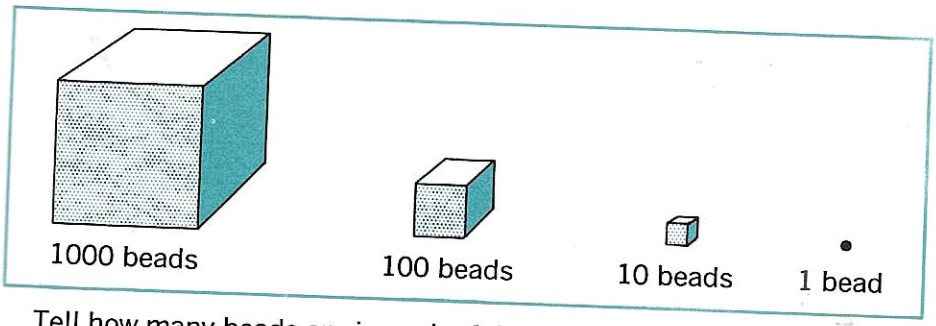


What will the odometer read after the car goes past 10 black markers?

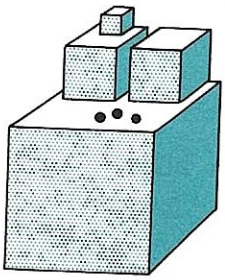
★ **6.** When Jim arrived home the odometer looked like this. → 13897

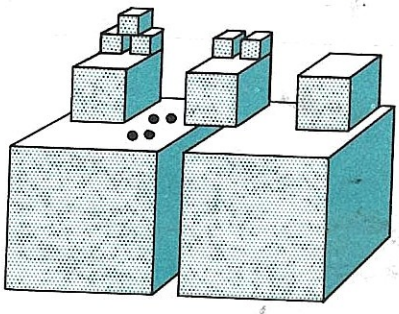
How far did Jim and his family travel on their vacation?

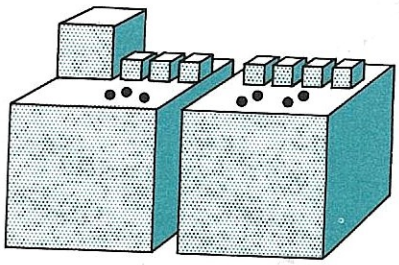
How many beads?

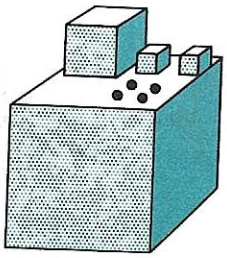


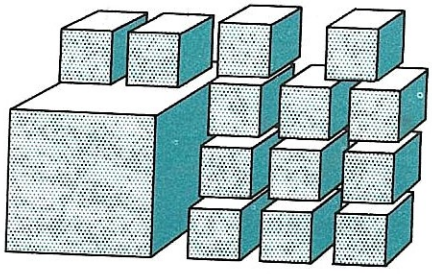
Tell how many beads are in each of these sets.

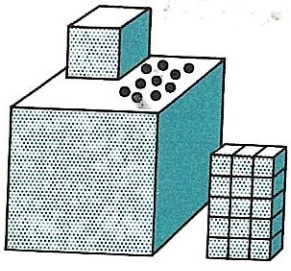
1. 

2. 

3. 

4. 

5. 

6. 

5- and 6-digit numerals

This digit tells how many thousands. → 7628

seven thousand six hundred twenty-eight

These digits tell how many thousands. → 36,584

thirty-six thousand five hundred eighty-four

These digits tell how many thousands. → 418,293

four hundred eighteen thousand two hundred ninety-three

ORAL EXERCISES

Read the number. Then tell how many thousands, hundreds, tens, and ones.

1. 8264	**7.** 58,403	**13.** 672,483	**19.** 601,485
2. 19,348	**8.** 27,064	**14.** 529,006	**20.** 902,209
3. 12,615	**9.** 152,634	**15.** 318,864	**21.** 333,333
4. 9485	**10.** 127,500	**16.** 902,902	**22.** 400,004
5. 24,652	**11.** 140,372	**17.** 587,418	**23.** 101,010
6. 91,860	**12.** 500,500	**18.** 70,070	**24.** 999,999

EXERCISES

Write the correct numeral for each number.

1. three hundred twenty-seven thousand two hundred sixteen
2. seventy-five thousand four hundred fifty-six
3. nine hundred thirty-seven thousand two hundred eighty-one
4. six hundred two thousand six hundred twenty
5. seven thousand seven
6. twenty-five thousand twenty-five
7. five hundred thousand five hundred
8. eight hundred thousand
9. one hundred eleven thousand two hundred twenty-two
10. three hundred fifty thousand ninety

One million

EXERCISES

1. Give the next 3 numbers for each of these.

[A]	[B]	[C]	[D]
500	950	5000	9500
600	960	6000	9600
700	970	7000	9700
.	.	.	.
.	.	.	.
.	.	.	.

[E]	[F]	[G]	[H]
50,000	95,000	500,000	950,000
60,000	96,000	600,000	960,000
70,000	97,000	700,000	970,000
.	.	.	.
.	.	.	.
.	.	.	.

2. Write the numeral for each of these.

[A] nine hundred thousand
[B] ten hundred thousand
[C] nine hundred ninety-nine thousand
[D] one thousand thousand

3. [A] What number is 1 more than 99?
[B] What number is 1 more than 999?
[C] What number is 1 more than 9999?
[D] What number is 1 more than 99,999?
[E] What number is 1 more than 999,999?

One million → 1,000,000

We can think of one million in different ways:

One million is a thousand thousand.

1,000,000

One million is ten hundred thousand.

1,000,000

7-, 8-, and 9-digit numerals

This digit tells how many millions. → 5,684,971

five million six hundred eighty-four thousand nine hundred seventy-one

These digits tell how many millions. → 56,742,314

fifty-six million seven hundred forty-two thousand three hundred fourteen

These digits tell how many millions. → 612,524,600

six hundred twelve million five hundred twenty-four thousand six hundred

ORAL EXERCISES

Read the numbers.

1. 2,364,827
2. 15,927,865
3. 327,786,934
4. 926,418,321
5. 75,642,872
6. 9,672,183
7. 37,618,425
8. 764,867,351
9. 761,804,700
10. 605,315,080
11. 700,600,300
12. 50,050,670
13. 18,000,000
14. 18,000,018
15. 600,000,000
16. 314,000,000
17. 726,800,000
18. 500,500,000
19. 73,006,043
20. 25,025,025
21. 250,250,250
22. 4,004,004
23. 999,999,999
24. 628,473,856

EXERCISES

1. In each part below, some of the digits are underlined. For these digits, tell how many thousands or how many millions.

[A] 268, 542, 716
[B] 413, 624, 981
[C] 76, 235, 651
[D] 82, 343, 612
[E] 5, 123, 426
[F] 78, 203, 654
[G] 56, 521, 000
[H] 93, 000, 000
[I] 93, 000, 000
[J] 482, 600, 000
[K] 482, 600, 000
[L] 700, 652, 000

2. Write the correct numeral for each number name.

[A] thirty-two million four hundred ninety-six thousand one hundred fifty-three

[B] six hundred million seven hundred eighty-three thousand nine hundred

[C] five million six hundred thousand one hundred fifty

[D] forty million three hundred fifty-six thousand seven hundred

[E] forty million three hundred fifty-six thousand twenty

[F] eight million five hundred thousand one hundred sixteen

[G] eight million seventy thousand one hundred sixteen

[H] six million six thousand six

3. Write the number that is

[A] one thousand more than 6,281,573.

[B] one million more than 6,281,573.

[C] one hundred thousand more than 6,281,573.

[D] two million more than 1,164,285.

[E] four thousand less than 164,285.

[F] sixty thousand less than 164,285.

[G] one hundred thousand less than 164,285.

[H] one million more than 164,285.

[I] one million less than 1,164,285.

4. Think carefully before you answer each question.

[A] How many hundreds make a thousand?

[B] How many thousands make a million?

5. For each pair of numbers below, write the larger one on your paper.

[A] 357,286	257,286	[G] 99,999,999	100,000,000
[B] 5,357,286	5,257,286	[H] 900,000,000	798,978,896
[C] 42,761	32,761	[I] 727,000,999	727,001,000
[D] 8,942,761	8,932,761	[J] 8,200,900	8,201,000
[E] 327,156,924	387,156,924	[K] 15,156,789	15,156,790
[F] 54,020,009	54,020,010	[L] 92,609,300	92,610,300

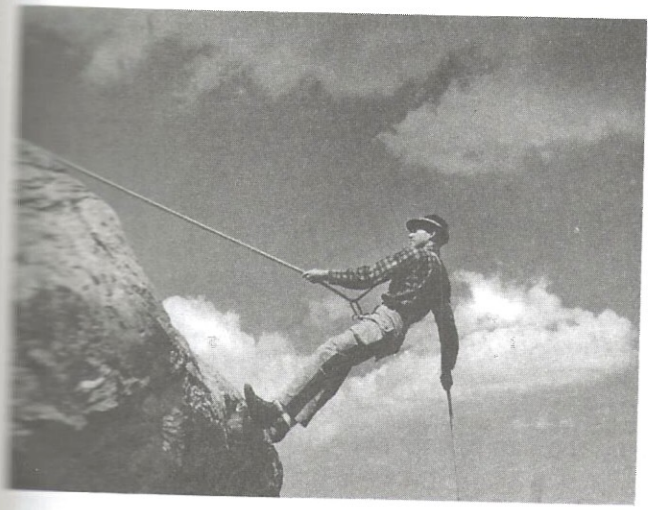
Mountain peaks in North America

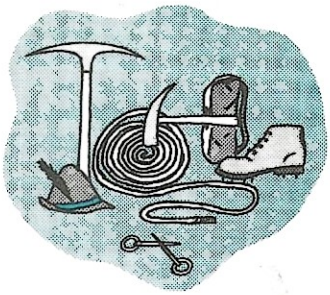
Mt. McKinley, Alaska

The first three mountains in the table are the highest in North America. There are other mountain peaks in North America higher than the remaining mountains listed in the table, but none is as high as the first three.

Name	Place	Feet
McKinley	Alaska	20,320
Logan	Canada	19,850
Citlaltepetl	Mexico	18,700
King	Canada	17,130
Steele	Canada	16,440
Bona	Alaska	16,420
Wood	Canada	15,880
Bear	Alaska	14,850
Whitney	California	14,495
Elbert	Colorado	14,431
Rainier	Washington	14,410
Lincoln	Colorado	14,284

Feet
0,320
9,850
3,700
7,130
6,440
5,420
5,880
4,850
4,495
4,431
4,410
4,284

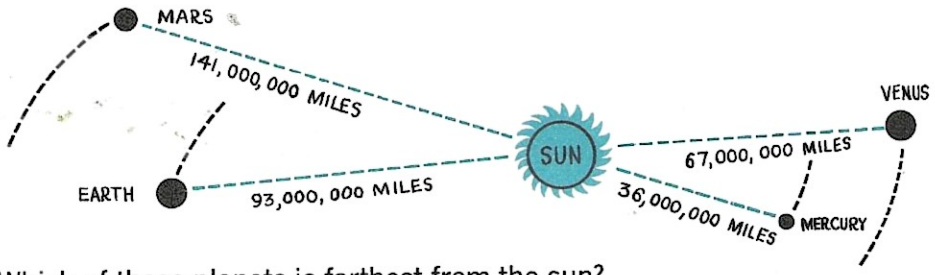




EXERCISES

1. What mountain peak in the table is more than 17,000 feet and less than 18,000 feet?
2. What mountain peak in the table is between 15,000 feet and 16,000 feet?
3. How many peaks in the table are less than 16,000 feet?
4. How many peaks in the table are more than 15,000 feet? How many are more than 14,000 feet?
5. If an airplane flies at 17,000 feet, over how many of these peaks could it fly?
6. Which mountain peaks are more than 14,400 feet and less than 14,500 feet?
7. How much higher is Mount Steele than Mount Bona?
8. How much higher is Mount Elbert than Mount Rainier?
9. Mount Hubbard (not given in the table) is 100 feet higher than Mount Bear. How high is Mount Hubbard?
10. How much higher is Mount Wood than Mount Bear?

Planets



1. Which of these planets is farthest from the sun?
2. Which of these planets is closest to the sun?
3. The distance of a planet from the sun is not always the same. How far would the earth be from the sun if it were a million miles closer than shown?
4. How far would the earth be from the sun if it were a million miles farther than shown?
5. Suppose Mars were ten million miles closer than shown. Then how far would it be from Mars to the sun?
6. How far would Mars be from the sun if it were 10,500,000 miles farther than shown?

One billion ⟶ 1,000,000,000

One billion is a thousand million.

1,000,000,000

One trillion ⟶ 1,000,000,000,000

One trillion is a thousand billion.

1,000,000,000,000

★ORAL EXERCISES

Read the numbers.

1. 52,387,268,491
2. 478,291,096,503
3. 7,836,403,728,500
4. 928,304,067,392,530

Chapter review

ORAL EXERCISES

Read the numbers.

1. 56,218
2. 275,654
3. 3,428,317
4. 86,379,435
5. 47,600
6. 547,600
7. 3,493,206
8. 567,285,431
9. 34,364,521
10. 176,349,821
11. 35,350,035
12. 600,000,000
13. 735,654
14. 8,008,008
15. 8,080,080
16. 8,800,800
17. 111,111,111
18. 7,654,392
19. 562,394
20. 2765
21. 901,318
22. 76,542,012
23. 201,302,405
24. 504,203,102

EXERCISES

1. Give the correct number for each of these.

[A] In 347, the 4 stands for [redacted]. (Answer: 40)
[B] In 6418, the 4 stands for [redacted].
[C] In 926, the 6 stands for [redacted].
[D] In 7286, the 7 stands for [redacted].
[E] In 48,354, the 8 stands for [redacted].
[F] In 68,024, the 0 stands for [redacted].
[G] In 4,373,654, the 7 stands for [redacted].
[H] In 5,718,304, the 5 stands for [redacted].
[I] In 123,619, the 2 stands for [redacted].
[J] In 305,651, the 0 stands for [redacted].
[K] In 27,354,683, the 2 stands for [redacted].
[L] In 528,764,318, the 5 stands for [redacted].

2. Write the correct numeral for each of these.

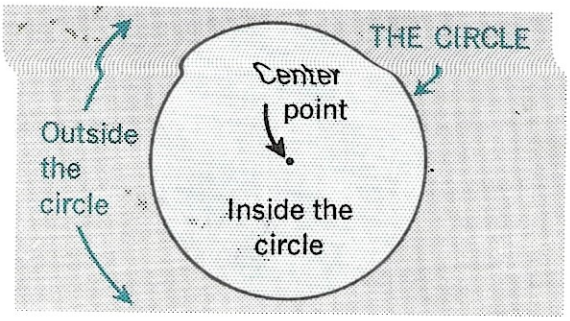
[A] six hundred twenty-eight thousand three hundred fifteen
[B] six million two hundred thirty-eight thousand
[C] seven hundred eighteen million
[D] five hundred million five hundred thousand five hundred
[E] eight million eight thousand eight

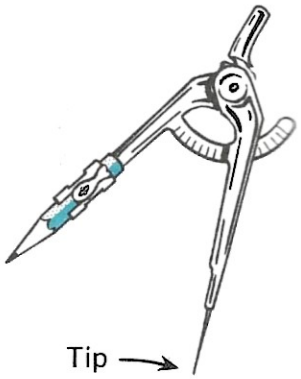
3. Write the correct numeral for each of these.

[A] 10 tens
[B] 10 hundreds
[C] *100 hundreds*
[D] 1000 tens
[E] 1000 hundreds
[F] *1000 thousands*

GEOMETRY - Lesson 1

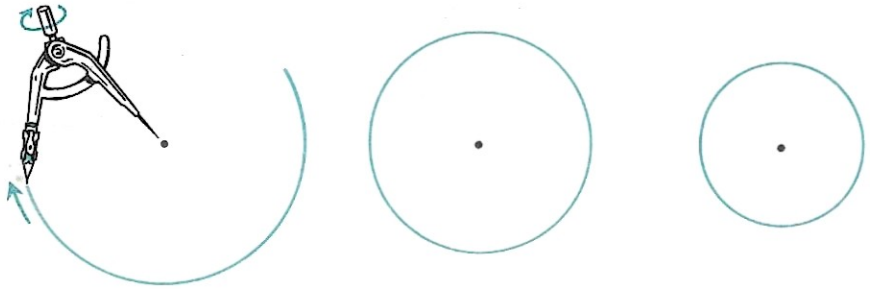
Circles and points



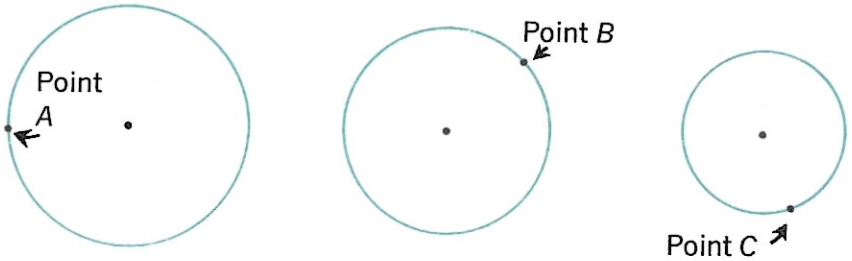


Here is a picture of a compass.

1. You can mark center points and use the compass to draw circles. Do this with your compass.



2. Points *A*, *B*, and *C* are marked on the circles below. Mark points on your circles and label them *D*, *E*, and *F*.



a compass.

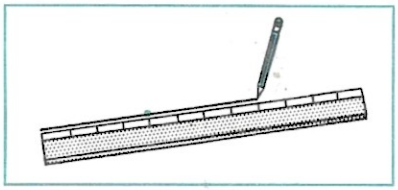
rcles.

3. Draw 5 circles of different sizes.

4. Mark a point on each circle. Label these points *A*, *B*, *C*, *D*, and *E*. It is easy to mark a **point** on a **circle**. Can you draw a **circle** through a **point**?

5. Mark a point *A* on your paper. Draw 6 different circles through this point.

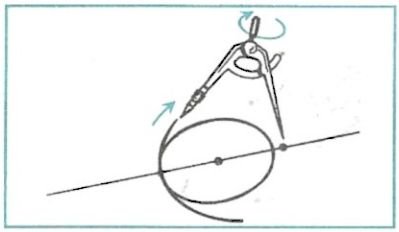
Here are some interesting ways to draw circles through points.



6. [a] Mark a point on your paper. Use your ruler to draw a line that goes through this point.

[b] Now draw 6 different-sized circles through the point. Do this by putting the tip of your compass at different places **on the line.**

[c] Do this again. This time make 2 of the circles the same size.

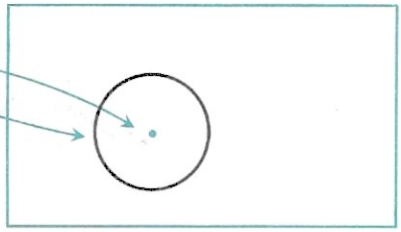


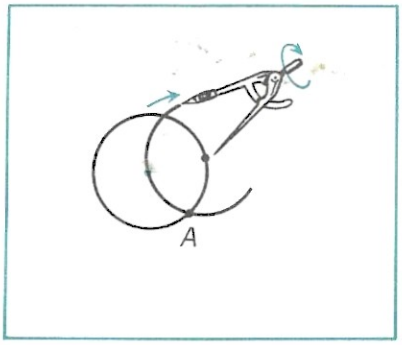
7. [a] Mark a point on your paper. Use your compass to draw a circle with this point as the center.

[b] Now draw 6 circles through this point. Do this by putting the tip of your compass at different places **on the circle.**

[c] Are all the circles the same size?

[d] Do this again. Each time, put the tip of your compass **where the circles meet** (for example, at point *A* in the picture).



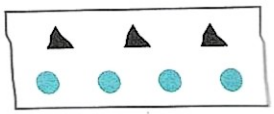


Chapter 2

ADDITION AND SUBTRACTION

Addition and subtraction are related

Sets help you understand addition and subtraction. Here is a set of triangles and circles.



Thinking about this set in different ways helps you understand these 4 equations.

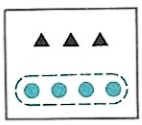
1. $3 + 4 = 7$
2. $4 + 3 = 7$
3. $7 - 4 = 3$
4. $7 - 3 = 4$

EXERCISES

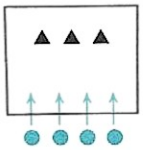
In the pictures below, when you see arrows, think about putting sets together. When you see objects in a dotted ring, think about removing them.

1. Match each equation above with the picture that explains it.

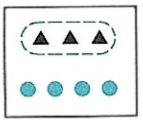
[A]



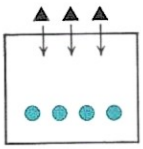
[B]



[C]



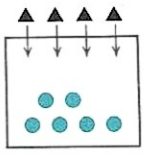
[D]



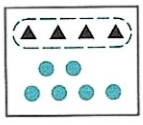
2. Match each picture with an equation.

[A] $4 + 6 = 10$ [B] $6 + 4 = 10$ [C] $10 - 6 = 4$ [D] $10 - 4 = 6$

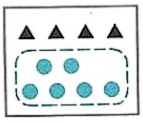
E



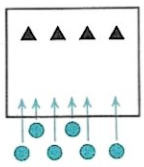
F



G



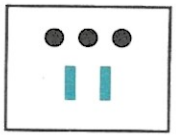
H



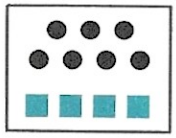
EXERCISES

1. Write 4 different equations for each set.

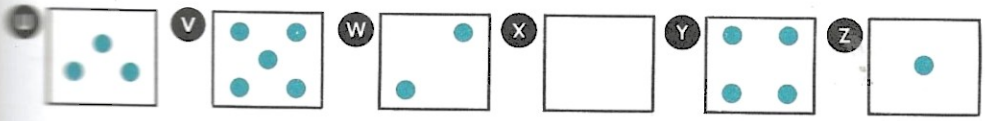
[A]

[B] 

[C]

[D] 

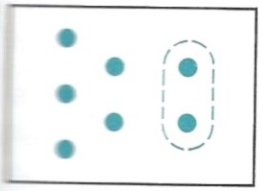
2. Answer the questions about the sets.

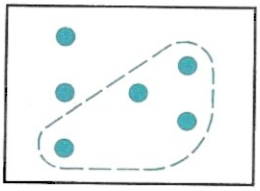


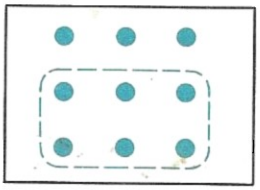
[A] How many dots in sets U and V together?
[B] How many dots in sets W and Y together?
[C] How many dots in sets V and Z together?
[D] How many dots in sets U and Y together?
[E] How many dots in sets W and X together?
[F] How many dots in sets V and X together?

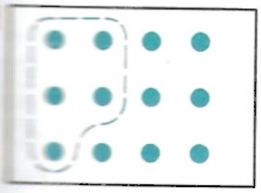
3. Write an addition equation for each part of exercise 2.

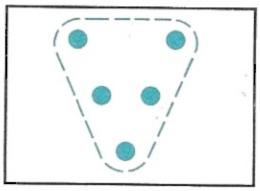
4. Think about removing the dots inside the dotted ring. Then write a subtraction equation for each set.

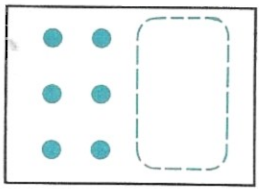
[A] 

[B] 

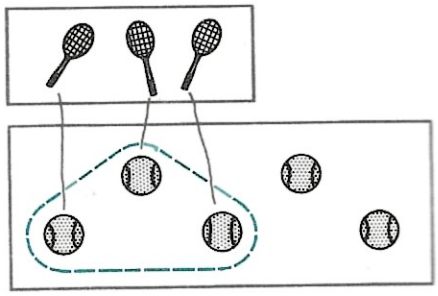
[C] 

[D] 

[E] 

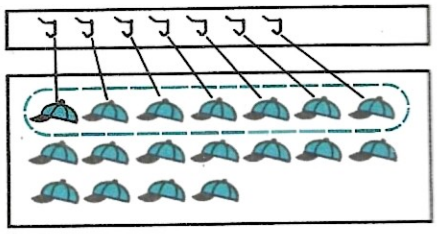
[F] 

Comparing sets



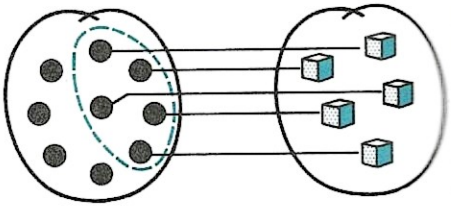
How many more tennis balls are there than rackets? You can draw matching lines to show that the answer is ▮. Also, you can think about subtraction.

$5 - 3 = \blacksquare$



How many more caps are there than hooks? By matching we see there are ▮ more caps than hooks. Here is the subtraction equation.

$18 - 7 = \blacksquare$

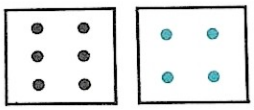


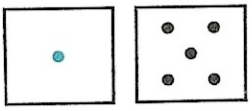
How many more balls are there than blocks? The matching lines show there are ▮ more balls than blocks. Also, you can use subtraction.

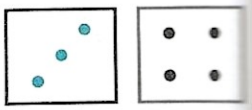
$9 - 5 = \blacksquare$

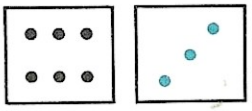
EXERCISES

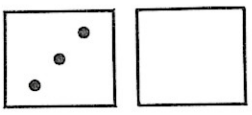
For each pair of sets, there are more black dots than blue dots. Write a subtraction equation for each pair to tell how many more black dots.

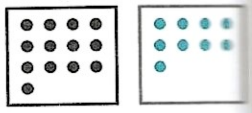
1. 

2. 

3. 

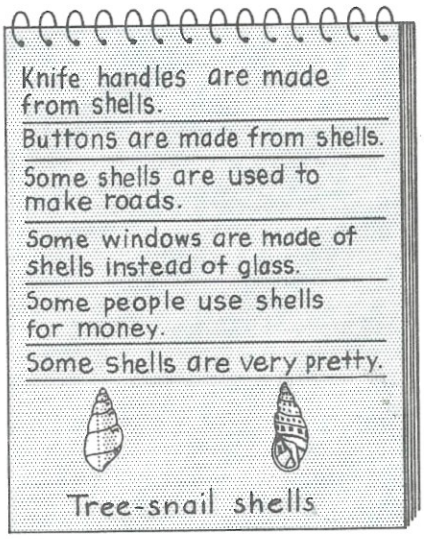
4. 

5. 

6. 

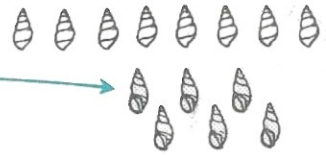
Collecting shells

Nan collects shells. She also collects facts about shells. Page 1 in her notebook looked like this.


Knife handles are made from shells.
Buttons are made from shells.
Some shells are used to make roads.
Some windows are made of shells instead of glass.
Some people use shells for money.
Some shells are very pretty.
Tree-snail shells

1. Nan collected 5 tree-snail shells and 3 clam shells. How many more tree-snail shells does she have than clam shells?

2. Nan and Jill went to the beach to look for shells. Nan placed her shells in a row like this. Jill placed her shells like this.



[A] Who had more shells? [B] How many more?

3. Nan found 9 mussel shells near the pond. She gave 4 of them to Jill. How many did Nan have left?

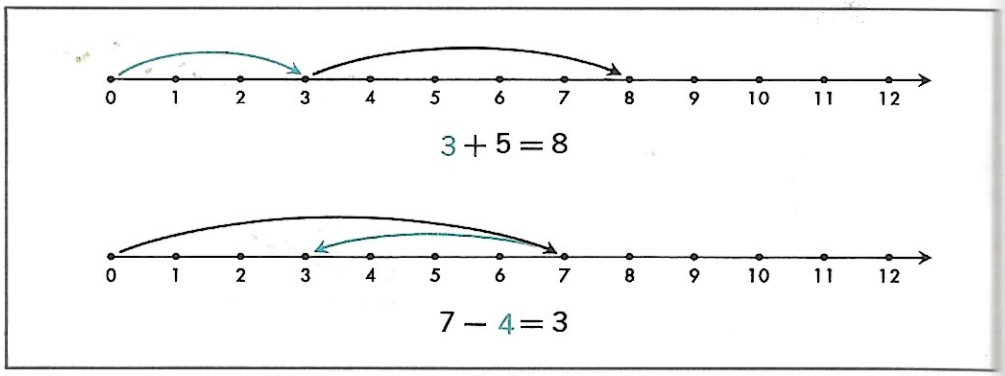
4. Nan wants pictures of her shells. She has 10 different snail shells. She has pictures of 7 of them. How many more pictures does she need?

5. Nan took 10 colored shells to school. She had 3 colored shells at home. She had fewer colored shells at home than at school. How many fewer?

*6. Nan picked out 5 white shells. She put some blue paint on 3 of them. Then she could see white on only 3 of the 5 shells. On how many could she see both blue and white?

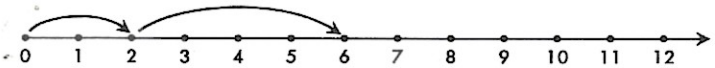
Addition and subtraction on the number line

The number line helps you think about addition and subtraction.

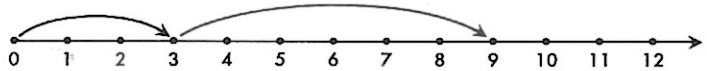


EXERCISES

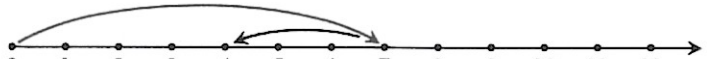
1. Give the missing number in the equation for each number line.

[A] 

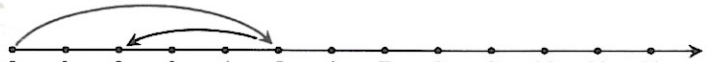
$2 + \square = 6$

[B] 

$\square + 6 = 9$

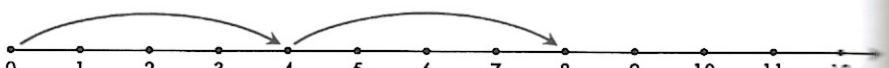
[C] 

$7 - \square = 4$

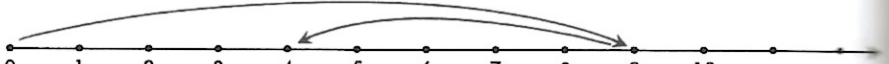
[D] 

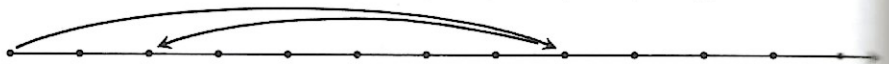
$\square - 3 = 2$

2. Give the correct equation for each number-line picture.

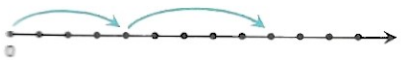
[A] 

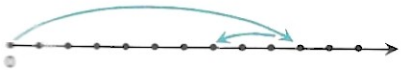
[B]

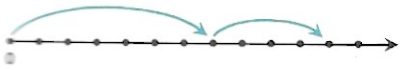
[C] 

[D] 

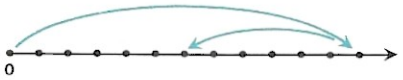
3. Write an equation for each number-line picture.

[A] 

[B] 

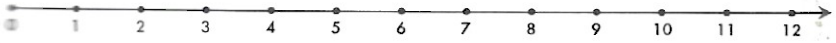
[C] 

[D]

[E] 

[F]

4. Draw a 6-inch line on your paper. Starting at the 0 dot, mark dots each half inch and label them as shown.



Now use arrows to show the sum $5 + 6$.

5. Draw a number line for each part below. Show the sum or difference by using arrows.

[A] $6 + 3$	[C] $11 - 9$	[E] $8 + 3$	[G] $11 - 3$
[B] $10 - 2$	[D] $12 - 2$	[F] $3 + 8$	[H] $6 + 4$

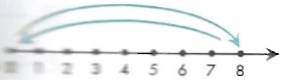
6. Write an equation for each part of exercise 5.

7. Draw a number line and show the sum $2 + 2 + 2 + 2 + 2 + 2$.

8. Draw a number line and show the sum $3 + 3 + 3 + 3$.

9. Write an equation for each number-line picture.

[A]

[B] 

Exercise for Experts

There is no largest number.
The last one can't be found.
Just add me to get the next.
There is no upper bound.

I'm a little bitty number.
So very small indeed.
If you add me to another,
A change you cannot read.

WHO AM I?

Addends and sums

The numbers we add are called addends. The answer is called the sum.

$4 + 3 = 7$

addend addend sum

$$\begin{array}{r} 2 \\ +\,6 \\ \hline 8 \end{array}$$

2 ← addend; +6 ← addend; 8 ← sum

Here are 3 ways to think about finding sums.

1. You can put sets together to tell how many there are in all.

$5 + 3 = 8$

addend addend sum

When one addend is 5 and the other addend is 3, the sum is 8.

2. You can use the number line or counting.

$4 + 2 = 6$

addend addend sum

3. You can use facts you already know.
Since $3 + 3 = 6$, you know that $3 + 4 = 7$.

EXERCISES

1. Give the sum for each gray square.

+	0	1	2	3	4	5	6	7	8	9
0							[A]			
1										
2						[B]			[C]	
3			[D]		[E]		[F]			
4							[G]			
5			[H]		[I]					
6	[J]			[K]						
7				[L]						
8		[M]								
9										

2. Find the sums.

[A] $\begin{array}{r} 6 \\ +3 \\ \hline \end{array}$ [B] $\begin{array}{r} 6 \\ +4 \\ \hline \end{array}$ [C] $\begin{array}{r} 5 \\ +4 \\ \hline \end{array}$ [D] $\begin{array}{r} 6 \\ +2 \\ \hline \end{array}$

[E] $\begin{array}{r} 7 \\ +3 \\ \hline \end{array}$ [F] $\begin{array}{r} 4 \\ +6 \\ \hline \end{array}$ [G] $\begin{array}{r} 4 \\ +3 \\ \hline \end{array}$ [H] $\begin{array}{r} 4 \\ +5 \\ \hline \end{array}$

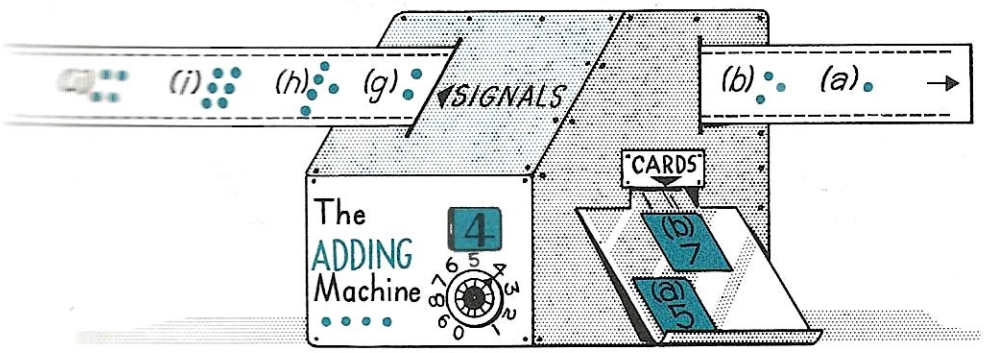
[I] $\begin{array}{r} 3 \\ +7 \\ \hline \end{array}$ [J] $\begin{array}{r} 5 \\ +5 \\ \hline \end{array}$ [K] $\begin{array}{r} 2 \\ +8 \\ \hline \end{array}$ [L] $\begin{array}{r} 5 \\ +2 \\ \hline \end{array}$

3. Solve the equations.

[A] $5 + 4 = \square$ [D] $\square + 4 = 9$

[B] $\square + 7 = 10$ [E] $\square + 6 = 10$

[C] $2 + \square = 8$ [F] $2 + \square = 9$



EXERCISES

1. Look at the machine and answer the questions.

   [a] What cards will the machine put out for signals (g), (h), (i), and (j)?

   [b] Suppose you change to an "Adding 2" machine. What cards will it put out for signals (g) through (j)?

   [c] What cards will an "Adding 3" machine put out for signals (g) through (j)?

   [d] What cards will an "Adding 8" machine put out for signals (g) through (j)?

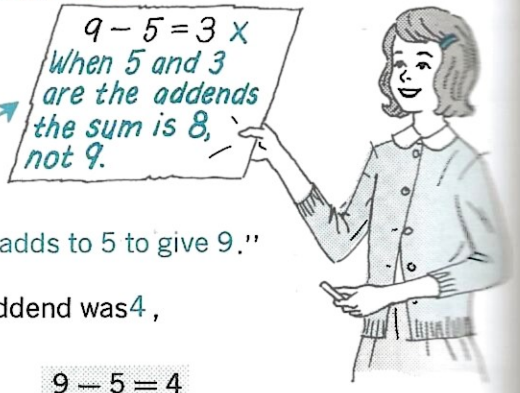
2. The "Adding 5" machine puts out [9]. What signal went into the machine?

3. Give what should go in each gray space in the table.

	Machine	Signal	Card
[A]	Adding 5	• • •	
[B]	Adding 5		8
[C]		• • •	8
[D]	Adding 3	⁘	
[E]	Adding 6	: :	
[F]	Adding 2		8
[G]	Adding 0	: : :	
[H]	Adding 0		3
[I]		: : :	10
[J]	Adding 6		9
[K]		⁙	10
[L]	Adding 4	• • •	

Differences and missing addends

Some students checked their arithmetic papers. Kris checked Jay's paper. Under one exercise, she wrote



Kris explained, "To find $9 - 5$, it helps to ask yourself what number adds to 5 to give 9."

When Jay found that the missing addend was 4, he wrote the difference correctly.

$$9 - 5 = 4$$

He said, "I know 4 is right because $4 + 5 = 9$."

Then Jay wrote this rule in his notebook:

We can find **DIFFERENCES** by looking for **MISSING ADDENDS.**

EXERCISES

1. Find the differences by finding the missing addends.

[A] $\square + 4 = 7$
$7 - 4 = \square$

[B] $\square + 3 = 10$
$10 - 3 = \square$

[C] $4 + \square = 9$
$9 - 4 = \square$

[D] $\square + 6 = 10$
$10 - 6 = \square$

[E] $4 + \square = 8$
$8 - 4 = \square$

[F] $\square + 4 = 5$
$5 - 4 = \square$

[G] $\square + 5 = 10$
$10 - 5 = \square$

[H] $6 + \square = 9$
$9 - 6 = \square$

[I] $\square + 1 = 7$
$7 - 1 = \square$

[J] $\square + 0 = 8$
$8 - 0 = \square$

[K] $7 + \square = 10$
$10 - 7 = \square$

[L] $3 + \square = 9$
$9 - 3 = \square$

2. Use addition facts to help you find these differences.

[A] $8 - 4 = \square$

[B] $10 - 9 = \square$

[C] $9 - 4 = \square$

[D] $7 - 4 = \square$

[E] $10 - 4 = \square$

[F] $10 - 8 = \square$

[G] $7 - 7 = \square$

[H] $7 - 1 = \square$

[I] $6 - 2 = \square$

[J] $9 - 7 = \square$

[K] $9 - 3 = \square$

[L] $5 - 2 = \square$

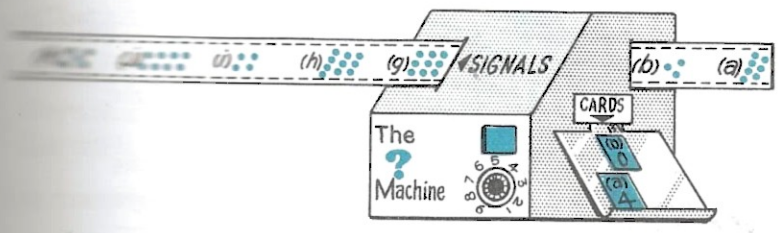
When you want to find the difference, think about finding the missing addend. You might think like this:

$$\begin{array}{r} 7 \\ -3 \\ \hline 4 \end{array}$$

Because $4 + 3 = 7$, I know 4 is the answer.

3. Find the differences by finding the missing addends.

[A] $\begin{array}{r} 10 \\ -6 \\ \hline \end{array}$	[B] $\begin{array}{r} 10 \\ -3 \\ \hline \end{array}$	[C] $\begin{array}{r} 8 \\ -2 \\ \hline \end{array}$	[D] $\begin{array}{r} 9 \\ -4 \\ \hline \end{array}$	[E] $\begin{array}{r} 4 \\ -2 \\ \hline \end{array}$	[F] $\begin{array}{r} 10 \\ -4 \\ \hline \end{array}$
[G] $\begin{array}{r} 6 \\ -3 \\ \hline \end{array}$	[H] $\begin{array}{r} 7 \\ -3 \\ \hline \end{array}$	[I] $\begin{array}{r} 5 \\ -5 \\ \hline \end{array}$	[J] $\begin{array}{r} 9 \\ -7 \\ \hline \end{array}$	[K] $\begin{array}{r} 5 \\ -2 \\ \hline \end{array}$	[L] $\begin{array}{r} 10 \\ -2 \\ \hline \end{array}$
[M] $\begin{array}{r} 9 \\ -3 \\ \hline \end{array}$	[N] $\begin{array}{r} 8 \\ -1 \\ \hline \end{array}$	[O] $\begin{array}{r} 6 \\ -0 \\ \hline \end{array}$	[P] $\begin{array}{r} 9 \\ -2 \\ \hline \end{array}$	[Q] $\begin{array}{r} 8 \\ -4 \\ \hline \end{array}$	[R] $\begin{array}{r} 5 \\ -3 \\ \hline \end{array}$
[S] $\begin{array}{r} 9 \\ -5 \\ \hline \end{array}$	[T] $\begin{array}{r} 10 \\ -7 \\ \hline \end{array}$	[U] $\begin{array}{r} 10 \\ -5 \\ \hline \end{array}$	[V] $\begin{array}{r} 10 \\ -8 \\ \hline \end{array}$	[W] $\begin{array}{r} 7 \\ -4 \\ \hline \end{array}$	[X] $\begin{array}{r} 7 \\ -2 \\ \hline \end{array}$



4. [A] What does this machine do?

[B] Show the cards the machine will put out for signals (g), (h), (i), (j), and (k).

[C] If this were a "Subtracting 2" machine, what cards would it put out for signals (a), (b), (g), (h), (i), (j), (k)?

5. Solve the equations.

[A] $7 - \square = 3$
[B] $\square - 6 = 2$
[C] $\square - 5 = 5$
[D] $\square - 4 = 4$
[E] $\square - 2 = 8$
[F] $9 - \square = 7$
[G] $10 - \square = 4$

Let's Review

- ADDITION
- SUBTRACTION
- MULTIPLICATION
- DIVISION
- MEASUREMENT
- PLACE VALUE
- INEQUALITIES
- PRINCIPLES

1. Find the sums and differences.

[A] $\begin{array}{r} 8 \\ +2 \\ \hline \end{array}$	[B] $\begin{array}{r} 6 \\ +3 \\ \hline \end{array}$	[C] $\begin{array}{r} 9 \\ -4 \\ \hline \end{array}$	[D] $\begin{array}{r} 7 \\ +1 \\ \hline \end{array}$
[E] $\begin{array}{r} 9 \\ -3 \\ \hline \end{array}$	[F] $\begin{array}{r} 6 \\ -5 \\ \hline \end{array}$	[G] $\begin{array}{r} 2 \\ +5 \\ \hline \end{array}$	[H] $\begin{array}{r} 5 \\ +5 \\ \hline \end{array}$
[I] $\begin{array}{r} 10 \\ -4 \\ \hline \end{array}$	[J] $\begin{array}{r} 7 \\ -3 \\ \hline \end{array}$	[K] $\begin{array}{r} 6 \\ +4 \\ \hline \end{array}$	[L] $\begin{array}{r} 10 \\ -1 \\ \hline \end{array}$
[M] $\begin{array}{r} 4 \\ +4 \\ \hline \end{array}$	[N] $\begin{array}{r} 7 \\ -5 \\ \hline \end{array}$	[O] $\begin{array}{r} 10 \\ -6 \\ \hline \end{array}$	[P] $\begin{array}{r} 0 \\ +6 \\ \hline \end{array}$
[Q] $\begin{array}{r} 4 \\ +5 \\ \hline \end{array}$	[R] $\begin{array}{r} 10 \\ -3 \\ \hline \end{array}$	[S] $\begin{array}{r} 10 \\ -7 \\ \hline \end{array}$	[T] $\begin{array}{r} 3 \\ +5 \\ \hline \end{array}$

2. [A] What number is 100 more than 9628?

[B] What number is 1000 less than 12,259?

[C] What number is 1,000,000 more than 7,265,348?

[D] What number is 1,000,000 less than 68,256,379?

[E] What number is 6000 more than 61,250?

[F] What number is 60,000 more than 1,000,000?

3. For each pair of numbers, write the larger one on your paper.

[A] 5,286,704
5,486,704

[B] 293,685,428
293,684,999

[C] 100,972,364
101,972,364

4. Write the correct numeral for each.

[A] sixty-eight million five hundred two thousand one hundred fifteen

[B] two hundred million eight hundred

[C] nine hundred thousand two

★**5.** Write the word name for 100,100,100.

Exercise for Experts

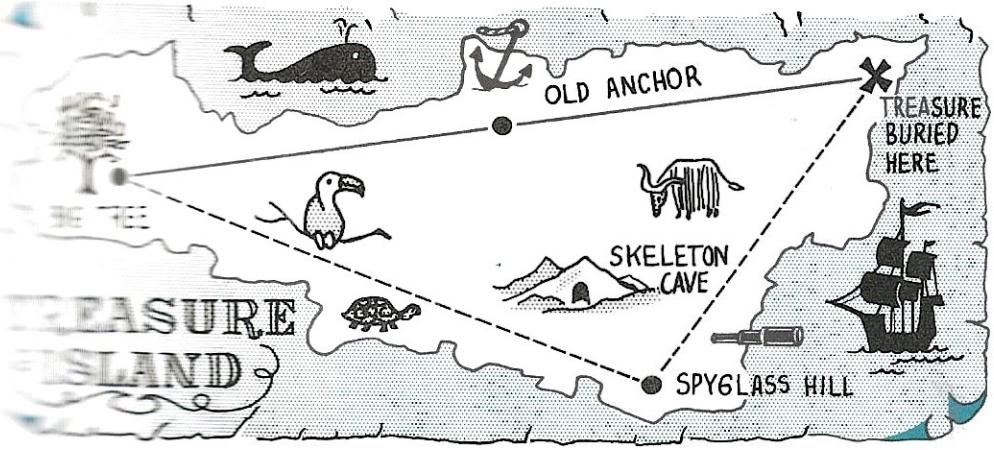
Addition is my dearest friend.
We never fuss or fight.
When I am done,
He comes and checks
To see that I am right.

I think I'm big until I spy
So many numbers larger than I.
My name has a one
And zeros galore.
Seven digits in all, not one more.

If I'm not right, I'll take no blame.
When I'm between,
They're just the same.

WHO AM I?

Treasure map



John found this map in an old trunk in his attic. One piece of paper in the trunk where the map was found looked like this.

Use scale A to answer these questions.

Scale A: 1 inch
Each inch on the map means 1 mile on the island.

1. How many miles is it from Big Tree to Spyglass Hill?
2. How many miles is it from Spyglass Hill to the treasure?

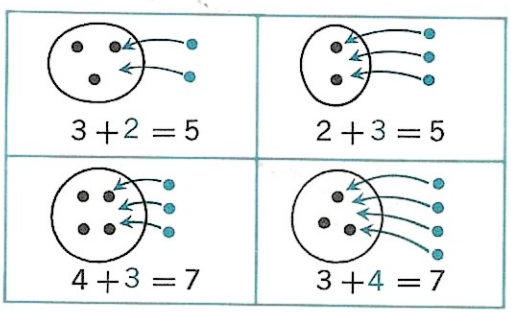
Scale B: 1 inch
Each inch on the map means 2 miles on the island.

There was another scrap of paper in the trunk. Use scale B to answer the rest of the questions.

3. How many miles is it from Big Tree to Spyglass Hill?
4. How many miles is it from Spyglass Hill to the treasure?
5. How many miles would you walk if you went from Big Tree to Spyglass Hill and then to the treasure?
6. How many miles is it from Big Tree straight to the treasure?
7. How many miles is it from Old Anchor straight to Skeleton Cave?
8. How far is it from Skeleton Cave to Spyglass Hill?
9. Which is farther from Big Tree, Spyglass Hill or the treasure? How much farther?

The order principle for addition

These 4 pictures help show that changing the order of two addends does not change the sum.



We call this idea:

The **ORDER** principle for addition.

EXERCISES

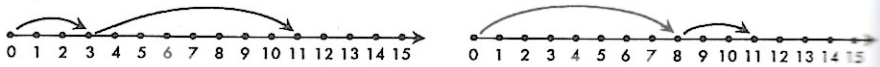
1. The number lines in exercise **A** show that $2 + 5 = 5 + 2$.
What do the number lines in exercises **B**, **C**, and **D** show?

[A]

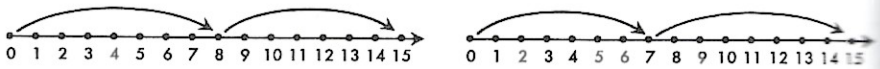
$$2 + 5 = 5 + 2$$

[B]

[C]



[D]



2. Solve the equations.

[A] $4 + 6 = 6 + \square$ [D] $7 + 9 = \square + 7$

[B] $5 + \square = 3 + 5$ [E] $327 + \square = 22 + 327$

[C] $\square + 8 = 8 + 9$ [F] $9856 + 6{,}542{,}349 = \square + 9856$

★ **3.** [A] Find a number $\square$ so that $7 + \square = \square + 7$.

[B] Is there more than one such number?

The grouping principle for addition

Think about 3 addends in a special order. 4 3 2

How shall we add if we do not change the order?

We could add these first.		We could add these first.
4 3 2	or	4 3 2
$(4 + 3) + 2$		$4 + (3 + 2)$
We think: $7 + 2$, the sum is 9.		We think: $4 + 5$, the sum is 9.

We may group the first two addends or the last two addends and still get the same sum.

We call this idea: The **GROUPING** principle for addition.

EXERCISES

A Find the sums by using the grouping shown. Do not change the order.

[C] 1 5 2 [E] 2 4 1 [G] 2 4 1

[D] 3 4 2 [F] 5 3 2 [H] 5 3 2

B The last two addends are grouped in the problems below. By changing the grouping, you can find the same sum more easily.

[D] $7 + (3 + 5)$	[E] $78 + (2 + 6)$	[I] $999 + (1 + 235)$
[E] $3 + (2 + 7)$	[F] $8 + (2 + 6)$	[J] $99 + (1 + 532)$
[F] $6 + (4 + 8)$	[G] $4 + (6 + 3)$	[K] $6 + (4 + 8321)$
[G] $25 + (1 + 7)$	[H] $5 + (5 + 3)$	[L] $7 + (3 + 8{,}267{,}529)$

C Solve these equations.

[A] $(5 + 3) + \square = 5 + (3 + 2)$ [B] $(\square + 29) + 67 = 48 + (29 + 67)$

Rearranging addends

Think about 3 addends. 2 4 3

When we change the **order** of the addends, we get the same sum.

When we **group** the addends differently, we get the same sum.

When we use these principles together, we begin by adding any two numbers.

We could add these first.	We could add these first.	We could add these first.
2 4 3	2 4 3	2 4 3
[A] $(2+4)+3$	[E] $2+(4+3)$	[I] $(2+3)+4$
[B] $(4+2)+3$	[F] $2+(3+4)$	[J] $4+(2+3)$
[C] $3+(2+4)$	[G] $(4+3)+2$	[K] $(3+2)+4$
[D] $3+(4+2)$	[H] $(3+4)+2$	[L] $4+(3+2)$

EXERCISES

1. Find each sum in the chart above.

2. Addends: 3 5 1
We could add the first two addends and write $(3+5)+1=9$.
Which addends do we add first when we write

[A] $5+(3+1)=9$? [C] $3+(5+1)=9$?

[B] $1+(5+3)=9$? [D] $(1+5)+3=9$?

3. Solve the equations.

[A] $(2+3)+5=$ ▮ [G] $5+(3+2)=$ ▮

[B] $2+(3+5)=$ ▮ [H] $(5+3)+2=$ ▮

[C] $2+(5+3)=$ ▮ [I] $(3+5)+2=$ ▮

[D] $(2+5)+3=$ ▮ [J] $3+(5+2)=$ ▮

[E] $(5+2)+3=$ ▮ [K] $3+(2+5)=$ ▮

[F] $5+(2+3)=$ ▮ [L] $(3+2)+5=$ ▮

Since we can change **order** and **grouping** and still get the same sum, we may **add any two** numbers first. We could think like this to find $6+3+4$.

$6+3+4$

Think: 10 and 3
Write: 13

or

6
3
4
13 ← Write

Think: 10 and 3

Find the sums. Look for tens.

[A] $7+3+2$	[D] $5+6+5$	[G] $6+3+1$	[J] $10+0+8$
[B] $7+2+3$	[E] $4+6+9$	[H] $5+2+8$	[K] $2+5+2$
[C] $9+8+1$	[F] $7+8+2$	[I] $8+5+2$	[L] $3+5+5$

Find the sums.

[A]	[B]	[C]	[D]	[E]	[F]	[G]	[H]	[I]
5	4	4	8	7	9	8	6	4
2	2	8	4	5	9	2	0	6
+4	+8	+2	+2	+3	+1	+8	+4	+9

Find the sums. Look for tens.

[A] $2+8+3+4$	[C] $2+3+4+8$	[E] $4+3+3+6$
[B] $2+3+8+4$	[D] $4+6+3+3$	[F] $7+4+3+5$

Find the sums.

[A]	[B]	[C]	[D]	[E]
9	8	5	6	6
7	2	7	4	7
1	3	3	7	4
+2	+1	+2	+3	+3

When we use the **order** and **grouping** principles together to find sums:

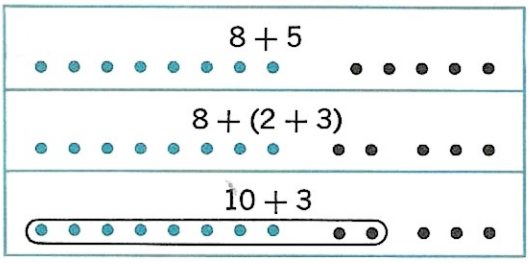
> We can **ARRANGE** the addends in any way and still get the same **SUM**.

Sums between 10 and 19

You can use what you have learned about grouping to find sums like these.

$8 + 5$
$7 + 4$
$9 + 6$

You can add 8 and 5 by looking for 10.



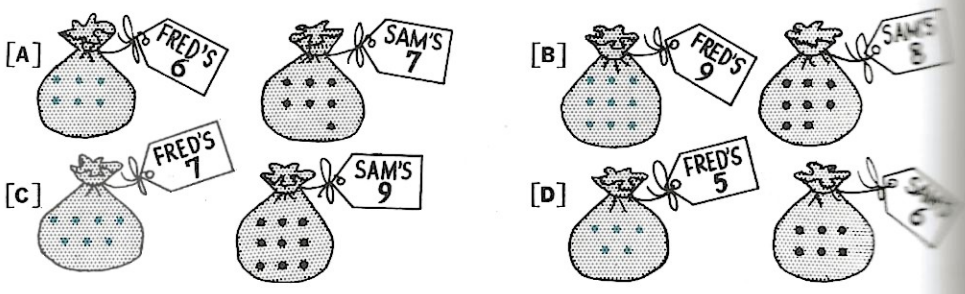
Explain why 5 is written as $2 + 3$ in this example.

$8 + 5$
$\downarrow$
$8 + 2 + 3$

Think: 10 and 3
Write: 13

EXERCISES

1. Tell how many of Sam's marbles Fred needs to make 10.



2. In each problem above, tell how many marbles Sam and Fred had in all. Do not count.

Solve the equations.

[A] $8 + \square = 10$ [D] $6 + \square = 10$ [G] $\square + 6 = 10$ [J] $\square + 4 = 10$
[B] $5 + \square = 10$ [E] $4 + \square = 10$ [H] $\square + 9 = 10$ [K] $\square + 5 = 10$
[C] $7 + \square = 10$ [F] $9 + \square = 10$ [I] $\square + 7 = 10$ [L] $\square + 8 = 10$

Solve the equations.

[A] $5 = \square + 2$ [D] $6 = \square + 2$ [G] $6 = 3 + \square$ [J] $8 = 4 + \square$
[B] $9 = \square + 4$ [E] $8 = \square + 5$ [H] $8 = 6 + \square$ [K] $5 = 3 + \square$
[C] $7 = \square + 4$ [F] $7 = \square + 2$ [I] $9 = 3 + \square$ [L] $6 = 1 + \square$

Find the missing numbers.

[A] $7 + 6 = 7 + (\square + 3)$ [G] $9 + 3 = 9 + (1 + \square)$
[B] $9 + 5 = 9 + (\square + 4)$ [H] $7 + 4 = 7 + (3 + \square)$
[C] $8 + 4 = 8 + (\square + 2)$ [I] $8 + 5 = 8 + (2 + \square)$
[D] $6 + 5 = 6 + (\square + 1)$ [J] $6 + 6 = 6 + (4 + \square)$
[E] $7 + 5 = 7 + (\square + 2)$ [K] $8 + 7 = 8 + (2 + \square)$
[F] $8 + 6 = 8 + (\square + 4)$ [L] $9 + 7 = 9 + (1 + \square)$

Find the missing numbers.

[A] $7 + 6 = 10 + \square$ [E] $7 + 5 = 10 + \square$ [I] $8 + 5 = 10 + \square$
[B] $9 + 5 = 10 + \square$ [F] $8 + 6 = 10 + \square$ [J] $6 + 6 = 10 + \square$
[C] $8 + 4 = 10 + \square$ [G] $9 + 3 = 10 + \square$ [K] $8 + 7 = 10 + \square$
[D] $6 + 5 = 10 + \square$ [H] $7 + 4 = 10 + \square$ [L] $9 + 7 = 10 + \square$

Find each sum by thinking first about how much needs to be added to the number to make 10.

[A] $3 + 3$ [D] $8 + 4$ [G] $8 + 5$ [J] $8 + 8$ [M] $3 + 9$ [P] $7 + 9$
[B] $5 + 2$ [E] $7 + 5$ [H] $9 + 4$ [K] $9 + 9$ [N] $5 + 7$ [Q] $6 + 8$
[C] $7 + 6$ [F] $9 + 6$ [I] $7 + 7$ [L] $7 + 4$ [O] $6 + 8$ [R] $5 + 6$

Exercise for Experts

I'm not so very big,
Just over 33.
Add some fives together,
My name you soon will see.

I'm a little over 16
And less than 22.
If you'll just start adding sixes,
My name will come to you.

WHO AM I?

Exercises

1. Find the missing numbers.

[A] $7+7=10+\square$
[B] $6+5=10+\square$
[C] $9+6=10+\square$
[D] $8+7=10+\square$
[E] $7+8=10+\square$
[F] $6+9=10+\square$
[G] $4+8=10+\square$
[H] $9+4=10+\square$
[I] $8+5=10+\square$
[J] $5+8=10+\square$
[K] $4+9=10+\square$
[L] $3+7=10+\square$

2. Find the sums.

[A] $\begin{array}{r}4\\+4\\\hline\end{array}$	[B] $\begin{array}{r}5\\+2\\\hline\end{array}$	[C] $\begin{array}{r}4\\+3\\\hline\end{array}$	[D] $\begin{array}{r}2\\+8\\\hline\end{array}$	[E] $\begin{array}{r}6\\+3\\\hline\end{array}$	[F] $\begin{array}{r}9\\+4\\\hline\end{array}$	[G] $\begin{array}{r}7\\+6\\\hline\end{array}$	[H] $\begin{array}{r}8\\+1\\\hline\end{array}$
[I] $\begin{array}{r}0\\+9\\\hline\end{array}$	[J] $\begin{array}{r}6\\+1\\\hline\end{array}$	[K] $\begin{array}{r}3\\+3\\\hline\end{array}$	[L] $\begin{array}{r}4\\+6\\\hline\end{array}$	[M] $\begin{array}{r}7\\+2\\\hline\end{array}$	[N] $\begin{array}{r}9\\+8\\\hline\end{array}$	[O] $\begin{array}{r}7\\+7\\\hline\end{array}$	[P] $\begin{array}{r}2\\+7\\\hline\end{array}$
[Q] $\begin{array}{r}4\\+2\\\hline\end{array}$	[R] $\begin{array}{r}6\\+6\\\hline\end{array}$	[S] $\begin{array}{r}5\\+5\\\hline\end{array}$	[T] $\begin{array}{r}8\\+3\\\hline\end{array}$	[U] $\begin{array}{r}7\\+8\\\hline\end{array}$	[V] $\begin{array}{r}6\\+7\\\hline\end{array}$	[W] $\begin{array}{r}5\\+3\\\hline\end{array}$	[X] $\begin{array}{r}6\\+8\\\hline\end{array}$

3. Read each exercise carefully. Then give the sum.

[A] Because $4+4=8$, we know that $4+5=\square$.
[B] Because $7+7=14$, we know that $7+6=\square$.
[C] Because $5+5=10$, we know that $5+6=\square$.
[D] Because $6+6=12$, we know that $6+7=\square$.
[E] Because $9+9=18$, we know that $9+8=\square$.
[F] Because $8+8=16$, we know that $8+9=\square$.
[G] Because $6+6=12$, we know that $6+5=\square$.
[H] Because $7+7=14$, we know that $6+8=\square$.
[I] Because $8+8=16$, we know that $7+9=\square$.
[J] Because $6+6=12$, we know that $5+7=\square$.

4. Give the sums.

[A] $\begin{array}{r}3\\4\\+5\\\hline\end{array}$
[B] $\begin{array}{r}5\\3\\+7\\\hline\end{array}$
[C] $\begin{array}{r}5\\1\\+9\\\hline\end{array}$
[D] $\begin{array}{r}5\\4\\+6\\\hline\end{array}$
[E] $\begin{array}{r}3\\2\\+6\\\hline\end{array}$
[F] $\begin{array}{r}8\\1\\+9\\\hline\end{array}$

5. Solve the equations.

[A] $3+2+4+5=\square$
[B] $1+2+3+4=\square$
[C] $(3+4)+(5+4)=\square$
[D] $(3+5)+(2+6)=\square$
[E] $\square+(3+6)=10$
[F] $8+\square=14$

differences when you know addends

You can find differences if you can find missing addends.

$$9 - 5 = \square \qquad \square + 5 = 9$$

These numbers are the same.

carefully. Then find the difference.

Because $6 + 7 = 13$, we know that $13 - 7 = \square$.
Because $8 + 7 = 15$, we know that $15 - 8 = \square$.
Because $9 + 5 = 14$, we know that $14 - 5 = \square$.
Because $8 + 6 = 14$, we know that $14 - 6 = \square$.
Because $9 + 7 = 16$, we know that $16 - 9 = \square$.

carefully. Then find the difference.

Because $48 + 37 = 85$, we know that $85 - 37 = \square$.
Because $76 + 88 = 164$, we know that $164 - 76 = \square$.
Because $57 + 19 = 76$, we know that $76 - 19 = \square$.
Because $95 + 83 = 178$, we know that $178 - 83 = \square$.

carefully. Then find the missing number.

To find $15 - 7$, it helps to think $7 + \square = 15$.
To find $17 - 9$, it helps to think $9 + \square = 17$.
To find $14 - 6$, it helps to think $6 + \square = 14$.
To find $12 - 7$, it helps to think $7 + \square = 12$.

$\square$ 12
$-\square$ -5
$=\square$

These numbers are the same.

Sometimes this is called the missing addend. This helps us think of subtraction as finding a missing addend.

4. Find the missing addends.

[A] $\begin{array}{r} 11 \\ -6 \\ \hline \end{array}$ [B] $\begin{array}{r} 12 \\ -4 \\ \hline \end{array}$ [C] $\begin{array}{r} 13 \\ -8 \\ \hline \end{array}$

[D] $\begin{array}{r} 13 \\ -9 \\ \hline \end{array}$ [E] $\begin{array}{r} 17 \\ -8 \\ \hline \end{array}$ [F] $\begin{array}{r} 14 \\ -9 \\ \hline \end{array}$

Another way to find differences

These exercises will help you think about another way to find differences.

EXERCISES

1. Solve the equations.

[A] $15 - 5 = \square$
[B] $17 - 7 = \square$
[C] $12 - 2 = \square$
[D] $13 - \square = 10$
[E] $11 - \square = 10$
[F] $15 - \square = 10$
[G] $13 - 3 = \square$
[H] $12 - \square = 10$
[I] $14 - \square = 10$
[J] $16 - \square = 10$
[K] $17 - \square = 10$
[L] $18 - \square = 10$

2. The numbers subtracted are given in blue. Make sure the answers are correct. For each exercise, tell how much was subtracted in all.

[A] $(15 - 5) - 2 = 8$
[B] $(17 - 7) - 2 = 8$
[C] $(12 - 2) - 3 = 7$
[D] $(15 - 5) - 1 = 9$
[E] $(17 - 7) - 2 = 8$
[F] $(14 - 4) - 4 = 6$
[G] $(13 - 3) - 4 = 6$
[H] $(16 - 6) - 3 = 7$
[I] $(16 - 6) - 1 = 9$

3. Solve the equations.

[A] $15 - 5 = \square$
$(15 - 5) - 2 = \square$
$15 - 7 = \square$

[B] $13 - 3 = \square$
$(13 - 3) - 3 = \square$
$13 - 6 = \square$

[C] $16 - 6 = \square$
$(16 - 6) - 1 = \square$
$16 - 7 = \square$

[D] $14 - 4 = \square$
$(14 - 4) - 5 = \square$
$14 - 9 = \square$

[E] $17 - 7 = \square$
$(17 - 7) - 2 = \square$
$17 - 9 = \square$

[F] $12 - 2 = \square$
$(12 - 2) - 6 = \square$
$12 - 8 = \square$

[G] $15 - 5 = \square$
$(15 - 5) - 4 = \square$
$15 - 9 = \square$

[H] $18 - 8 = \square$
$(18 - 8) - 1 = \square$
$18 - 9 = \square$

[I] $13 - 3 = \square$
$(13 - 3) - 5 = \square$
$13 - 8 = \square$

You can find differences such as $15 - 7$ by doing two easy subtractions in place of a harder one.

$$\begin{array}{r} 15 \\ -5 \\ \hline 10 \\ -2 \\ \hline 8 \end{array}$$

$15 - 7$ ← Subtracting 7 → $(15 - 5) - 2 = 8$

$15 - 7 = 8$

Finding sums and differences

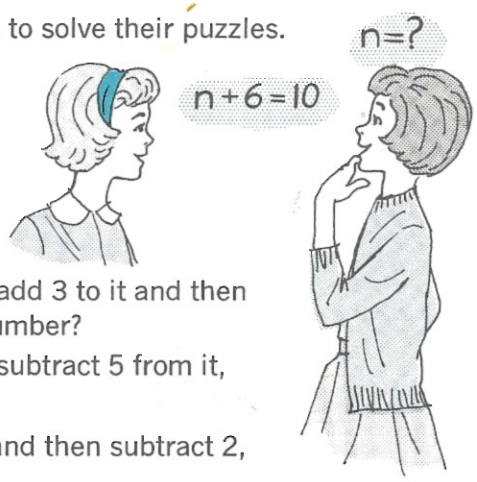
1. Find the differences.

[A] $12-4=\square$ [D] $14-5=\square$ [G] $16-9=\square$ [J] $17-8=\square$
[B] $11-5=\square$ [E] $14-7=\square$ [H] $13-6=\square$ [K] $12-7=\square$
[C] $13-6=\square$ [F] $15-8=\square$ [I] $11-8=\square$ [L] $13-9=\square$

2. Find the sums and differences. Use any method you choose.

[A] 8 $\underline{+2}$	[B] 9 $\underline{-6}$	[C] 8 $\underline{+7}$	[D] 7 $\underline{+6}$	[E] 12 $\underline{-3}$	[F] 14 $\underline{-6}$	[G] 7 $\underline{+8}$	[H] 8 $\underline{+0}$
[I] 13 $\underline{-4}$	[J] 13 $\underline{-5}$	[K] 6 $\underline{+6}$	[L] 17 $\underline{-7}$	[M] 18 $\underline{-9}$	[N] 6 $\underline{+8}$	[O] 5 $\underline{+5}$	[P] 7 $\underline{+3}$
[Q] 10 $\underline{-4}$	[R] 10 $\underline{+8}$	[S] 9 $\underline{+7}$	[T] 15 $\underline{-7}$	[U] 8 $\underline{+8}$	[V] 9 $\underline{+9}$	[W] 16 $\underline{-6}$	[X] 14 $\underline{-9}$

3. Jane and Brenda played a game. Try to solve their puzzles.



[A] I'm thinking of a number. If you add 6 to it, you get 10. What is the number?

[B] I'm thinking of a number. If you add it to 4, you get 8. What is the number?

[C] I'm thinking of a number. If you add 3 to it and then add 2, you get 10. What is the number?

[D] I'm thinking of a number. If you subtract 5 from it, you get 10. What is the number?

[E] If you subtract 6 from a number and then subtract 2, you get 8. What is the number?

[F] I'm thinking of a number. If you add it to itself and then add 4, you get 10. What is the number?

4. Solve the equations.

[A] $14-\square=6$ [D] $\square-6=9$ [G] $\square-9=4$ [J] $10-\square=7$
[B] $5+8=\square$ [E] $\square+6=11$ [H] $6+\square=10$ [K] $2+\square=10$
[C] $17-\square=10$ [F] $8+\square=15$ [I] $\square+8=17$ [L] $\square+9=13$

Short stories

1. 11 balloons.
   3 blew away. How many left?

2. 5 girls with balloons.
   9 girls without.
   How many girls?

3. Jumping contest.
   9 grasshoppers. 7 crickets.
   How many in all?

4. 5 merit badges.
   Need 14 in all.
   How many more needed?

5. 13 wins.
   6 losses. How many more wins than losses?

6. 13 balloons.
   Stick 8 with a pin.
   How many left?

7. 8 cats. 13 monkeys. How many more monkeys than cats?

8. 9 points scored. 16 points needed to win the game. How many more points must be scored?

9. 10 fingers. 10 toes. How many more fingers than toes?

10. 16 flies. 7 frogs.
    Each frog gets a fly.
    How many flies are left?

11. 8 girls. 8 shoes. How many more shoes do they need?

12. 12 peanuts.
    Ate 2 and drank milk. Ate 5 more. How many left?

13. 12 peanuts.
    Ate 2 and drank milk. Ate 5 more. How many eaten in all?

14. Caught 17 fish.
    9 too small so threw them back.
    How many left?

15. 14 boys.
    6 hats.
    How many fewer hats than boys?

16. Had 15 cents. Spent 5 cents, and then spent 3 cents. How much left? How much spent?

17. Caught 12 butterflies. 7 got away. How many left?

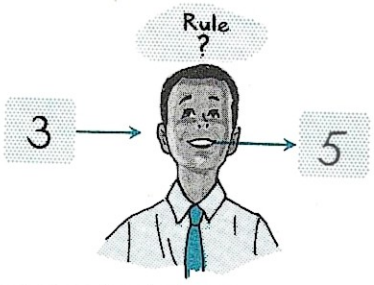
18. 17 papers.
    Want to sell all but 10.
    How many must be sold?

19. Gave 8 valentines.
    Received 17 valentines. Gave how many fewer than received?

20. 13 newspapers. Sold 3 to women, and then sold 4 to men.
    How many customers?
    How many papers left?

Guess-the-rule games

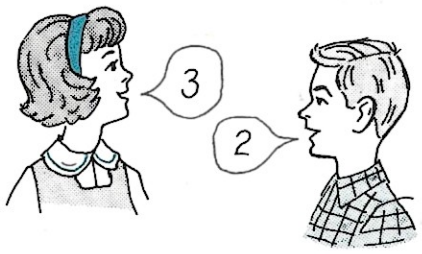
When Jane said 3, Tom answered 5.
When Jane said 8, Tom answered 10.
When Jane said 5, Tom answered 7.
When Jane said 2, what did Tom answer?
What rule do you think Tom was using?



Sue thought of a rule.
When Dick said 2, Sue answered 4.
When Dick said 5, Sue answered 10.
Dick said 9. What did Sue answer?
What rule do you think Sue was using?

Dick's number		Sue's answer
2 →	rule ???	→ 4
5 →		→ 10
9 →		→ ?

When Nancy said 3, Mike answered 2.
When Nancy said 8, Mike answered 2.
When Nancy said 12, Mike answered 2.
When Nancy said 0, Mike answered 2.
What do you think Mike answered when Nancy said 50? What rule do you think Mike was using?



EXERCISES

Study the tables carefully. Guess the rule, then give what you think should go in each ▩.

1. Carol's number / Jill's answer

4	14
6	16
2	12
7	▩
9	▩
▩	13
24	▩

2. Cindy's number / Nan's answer

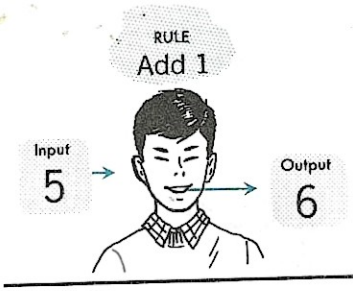
	8	1
	7	0
	10	3
[A]	9	▩
[B]	▩	6
[C]	▩	8
[D]	▩	10

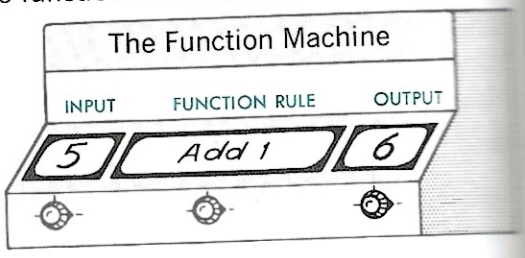
★ **3.** Cathy's number / Susan's answer

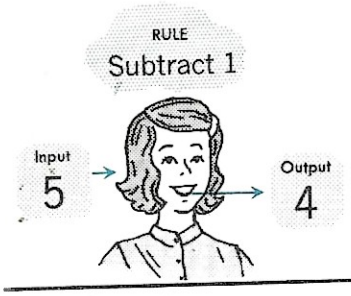
	2	0
	4	0
	3	1
	5	1
	8	0
[A]	6	▩
[B]	11	▩

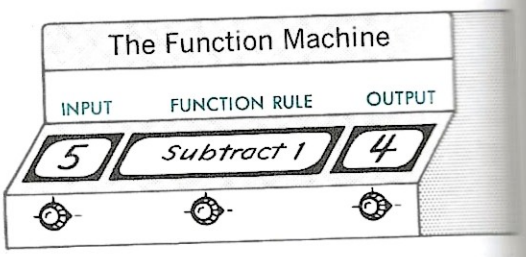
The function game

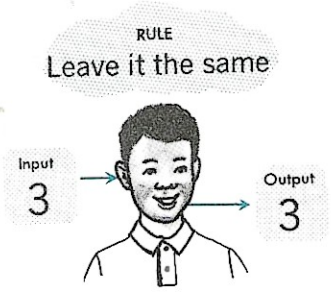
Study the pictures to see how the function machine works.

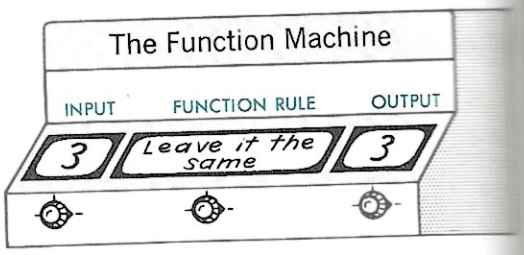


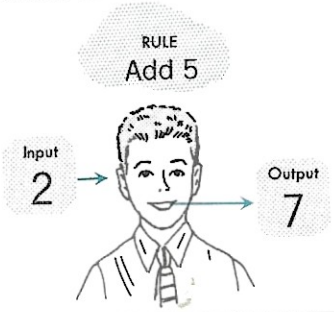


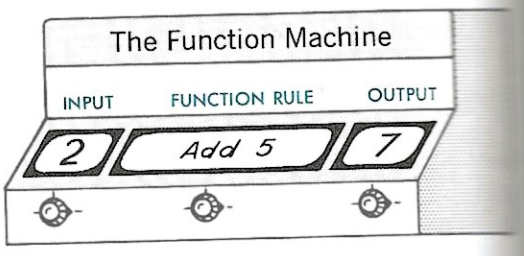












EXERCISES

Think about the function machine and tell what you think should go in each gray space.

1. Function Rule: Add 5

	Input	Output
	2	7
	4	9
[A]	1	
[B]	5	
[C]	3	

2. Function Rule: Subtract 3

	Input	Output
[A]	6	
[B]	8	
[C]	10	
[D]	3	
[E]	13	

3. Function Rule

	Input	Output
[A]		
	2	6
	4	8
	7	11
[B]	8	
[C]		13

4. Function Rule: Subtract 2

	Input	Output
	3	1
[A]	7	
[B]		3
[C]	10	
[D]		10

5. Function Rule

	Input	Output
[A]		
	7	1
	10	4
	6	0
[B]	9	
[C]		8

6. Function Rule: Subtract 8

	Input	Output
	10	2
[A]	13	
[B]	15	
[C]		3
[D]	17	

7. Function Rule

	Input	Output
[A]		
	2	10
	5	13
	7	15
[B]	3	
[C]		17

8. Function Rule

	Input	Output
[A]		
	0	9
	1	10
	9	18
[B]	7	
[C]	5	

9. Function Rule: If **odd**, subt. 1 If **even**, add 0

	Input	Output
	3	2
	2	2
	5	4
[A]	9	
[B]	8	

Let's Review

- ■ ADDITION
- SUBTRACTION
- MULTIPLICATION
- DIVISION
- MEASUREMENT
- ■ PLACE VALUE
- ■ INEQUALITIES
- PRINCIPLES

2. Write the missing numerals.

[A] For 3 tens and 6, we write ▪.
[B] For 5 tens and 0, we write ▪.
[C] For 1 ten and 8, we write ▪.
[D] For 0 tens and 8, we write ▪.
[E] 60 means 6 tens and ▪.
[F] For 1 ten and 0, we write ▪.
[G] For 7 tens and 4, we write ▪.
[H] 28 means 2 tens and ▪.
[I] 46 means ▪ tens and 6.

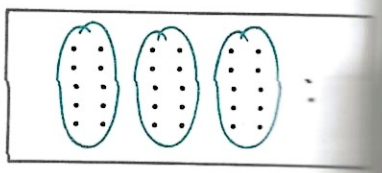
3. Give the missing words.

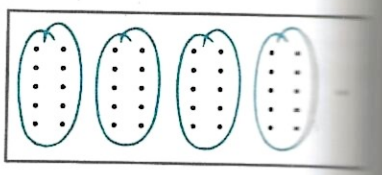
[A] In 1847, the 4 means four_?_.
[B] In 6253, the 6 means six_?_.
[C] In 2584, the 5 means five_?_.

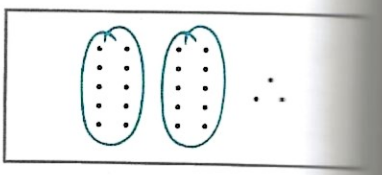
4. Solve the equations.

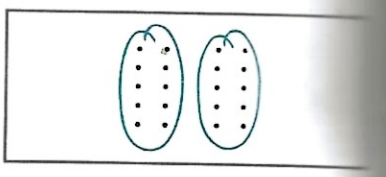
[A] $63 = 60 + \square$
[B] $10 + 7 = \square$
[C] $60 + 2 = \square$
[D] $18 = \square + 8$
[E] $70 = 70 + \square$
[F] $15 = 10 + \square$
[G] $80 + 0 = \square$
[H] $30 + 4 = \square$
[I] $67 = \square + 7$
[J] $60 + \square = 65$

1. In the pictures below there are 10 dots in each ring. Without counting, give the number of dots in each box.

[A] 

[B] 

[C] 

[D] 

5. In each pair, which number is larger?

[A] 62,429
62,197

[B] 28,315
30,000

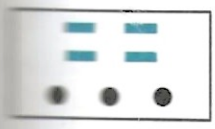
[C] 126,418
109,348

[D] 9999
10,000

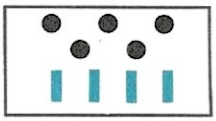
[E] 6,354,681
6,364,681

Chapter review

1. Write 2 addition and 2 subtraction equations for each set.

[A] 

[B]

[C] 

2. Find the sums and differences.

[A]	[B]	[C]	[D]	[E]	[F]	[G]	[H]
8	6	9	2	10	7	6	4
$+2$	$+3$	-4	$+5$	-4	-3	$+4$	$+4$

[I]	[J]	[K]	[L]	[M]	[N]	[O]	[P]
7	8	0	8	10	4	10	3
$+5$	-5	$+6$	-2	-6	$+5$	-3	$+5$

3. Find the differences.

[A] $7-3=$ ▢
[B] $13-9=$ ▢
[C] $11-3=$ ▢
[D] $15-6=$ ▢
[E] $15-7=$ ▢

[F] $12-9=$ ▢
[G] $16-8=$ ▢
[H] $13-6=$ ▢
[I] $14-7=$ ▢
[J] $13-5=$ ▢

4. Find the sums. Look for a sum of 10.

[A] $6+4+7$
[B] $3+7+3$
[C] $8+9+2$
[D] $5+7+5$
[E] $4+6+5$

[F] $7+8+3$
[G] $6+2+8$
[H] $9+8+1$
[I] $6+7+4$
[J] $1+9+5$

5. Find the missing numbers.

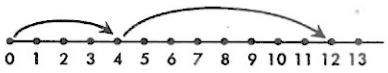
[A] $47+38=85$, then $85-38=$ ▢.
[B] $\square-18=74$, then $74+18=$ ▢.
[C] $37+\square=68+$ ▢
[D] $(39+27)+68=39+(\square+68)$

6. Should the ● be $+$ or $-$?

[A] $(5-2)=(9-5)$ ● 2
[B] $(\square+3)=(12-8)$ ◍ 3

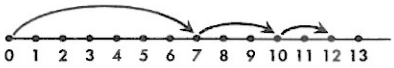
7. Use the number line to help you solve each equation.

[A]



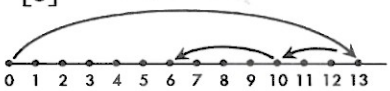
$4+\square=12$

[B]



$7+\square+2=12$

[C]



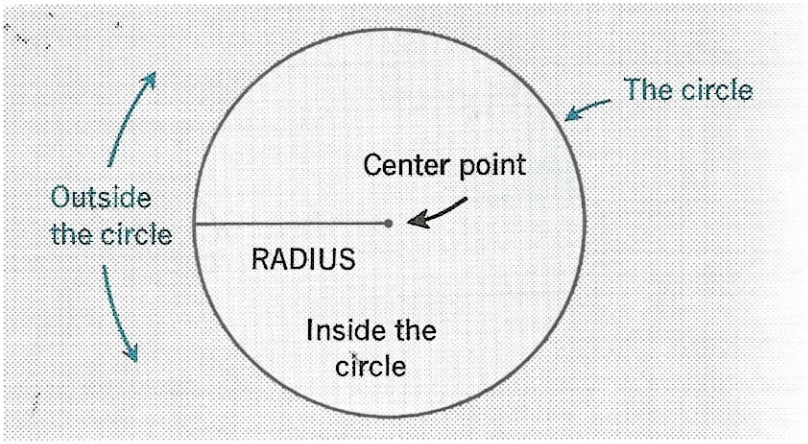
$(13-3)-\square=6$

Exercise for Experts

Increase me by 5.
Then take away 7.
When you are done,
You should have 11.
WHO AM I?

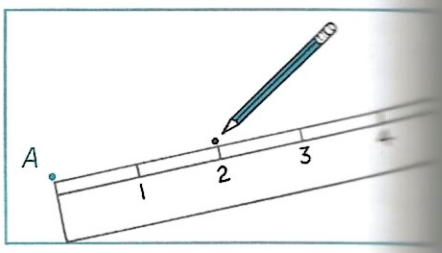
GEOMETRY - Lesson 2

Circles and pairs of points

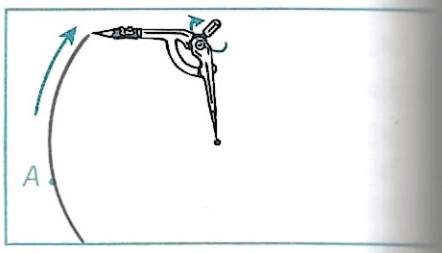


This circle has a 1-inch radius.

1. [A] Mark point *A* on your paper. Then mark a second point 2 inches from *A*.



[B] Now use your compass to draw a circle through point *A*. Your circle should have a **2-inch radius.**



2. Mark points *B*, *C*, and *D* on your paper.

[A] Draw a circle with a 3-inch radius that goes through point *B*.

[B] Draw a circle with a $1\frac{1}{2}$-inch radius that goes through point *C*.

[C] Draw a 6-centimeter radius that goes through point *D*.

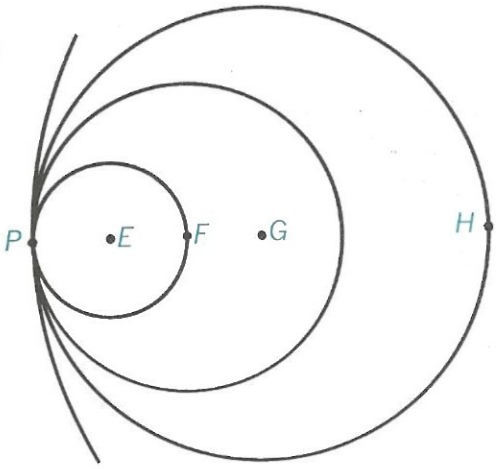
Use your centimeter ruler to measure

a) the radius of the circle with center E,

b) the radius of the circle with center F,

c) the radius of the circle with center G,

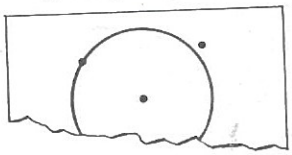
d) the radius of a circle that has center H and goes through point P.



It is easy to draw a circle through **one** point. Can you draw a circle that goes through **two** points?

Draw 2 points on your paper. Try to draw some circles that go through both points.

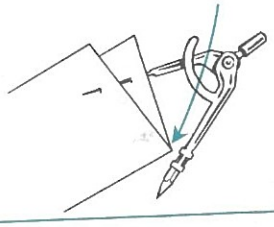
If your drawing looks like this, you are close but not close enough.

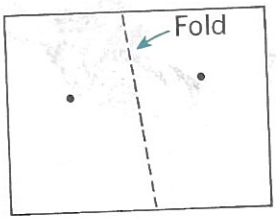


Here is a way to draw a circle through 2 points without guessing.

Put the compass through **both points.**

Fold carefully





5. Draw 2 points on your paper. Draw a circle through the points as shown above. Put the tip of your compass at different places on the circle, and draw 10 different-sized circles through the 2 points.

6. Draw the circle that goes through the 2 points of exercise 5 and has the smallest possible radius. Where did you put the tip of your compass?